Period 3, Nov 20, 2024

1) $y = \sin 3x \Rightarrow \frac{dy}{dx} = \underline{(\cos 3x) \cdot 3}$

2) $y = (x^2 - 5x)^3 \Rightarrow \frac{dy}{dx} = \underline{3(x^2-5x)^2(2x-5)}$

3) $y = \sqrt{(7-6x^3)^5} \Rightarrow \frac{dy}{dx} = \frac{5}{2}(7-6x^3)^{3/2}(-18x^2)$

$(7-6x^3)^{5/2}$

$y = \sin 3x \Rightarrow y = \sin u$

$u = 3x$

$\frac{du}{dx} = 3$

$\frac{dy}{du} = \cos u$

$\cancel{\frac{du}{dx}} \cdot \cancel{\frac{dy}{du}} = \frac{dy}{dx}$

$3 \cdot \cos u = \frac{dy}{dx}$

$3 \cdot \cos 3x = \frac{dy}{dx}$

$y = (x^2-5x)^3 \Rightarrow y = u^3$

$u = x^2 - 5x$

$\frac{du}{dx} = 2x - 5$

$\frac{dy}{du} = 3u^2$

$\cancel{\frac{du}{dx}} \cdot \cancel{\frac{dy}{du}} = \frac{dy}{dx}$

$(2x-5) \cdot 3 \cdot u^2 = \frac{dy}{dx}$

$(2x-5)(3)(x^2-5x)^2 = \frac{dy}{dx}$

$y = (7-6x^3)^{5/2} \Rightarrow y = u^{5/2}$

$u = 7 - 6x^3$

$\frac{du}{dx} = -18x^2$

$\frac{dy}{du} = \frac{5}{2}u^{\frac{3}{2}}$

$\cancel{\frac{du}{dx}} \cdot \cancel{\frac{dy}{du}} = \frac{dy}{dx}$

$(-18x^2)\left(\frac{5}{2}u^{3/2}\right) = \frac{dy}{dx}$

$\frac{5}{2}(-18x^2)(7-6x^3)^{3/2} = \frac{dy}{dx}$

Take der "with Respect To Time"

a) $a^2 + b^2 = c^2$

$2a \cdot \frac{da}{dt} + 2b \cdot \frac{db}{dT} = 2c \frac{dc}{dT}$

b) $A = \pi r^2$

$\frac{dA}{dT} = 2 \cdot \pi \cdot r \cdot \frac{dr}{dT}$

c. $Tan\,\theta = \frac{x}{5} \Rightarrow \sec^2\theta \cdot \frac{d\theta}{dT} = \frac{1}{5} \cdot \frac{dx}{dT}$

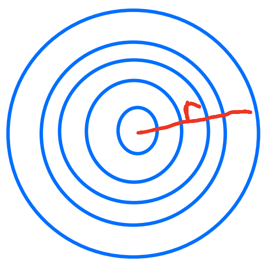
d. $2y^2 - 3x^4 = 0$

$4y \cdot \frac{dy}{dT} - 12x^3 \frac{dx}{dT} = 0$

Example 5: A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius *r* of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area *A* of the disturbed water changing?

$\frac{dA}{dt}$ = Rate of change or Area with respect to time

The Rate of change of radius = $\frac{dr}{dT}$



$$A = \pi r^2$$

$$\frac{dA}{dT} = \pi \cdot 2r \cdot \frac{dr}{dT}$$

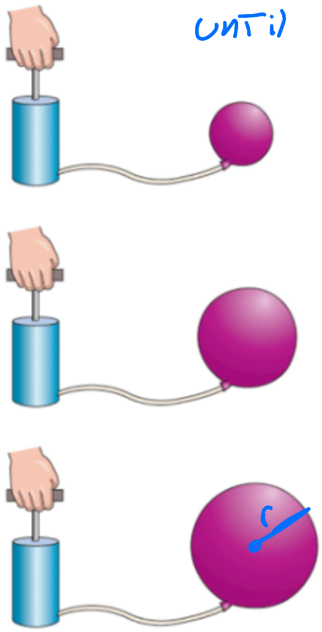
$$\frac{dA}{dt} = \pi \cdot 2 \cdot 4\,FT \cdot 1\,FT/sec$$

$$\frac{dA}{dT} = 8\pi \cdot FT^2/sec$$

Example 6: Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet.

$\frac{dV}{dt} = \frac{9}{2}\,\frac{FT^3}{min}$ $\qquad \frac{dr}{dT} = ?$

$r = 2$ until the end



Given:

Want: $V = \frac{4}{3}\pi r^3$

Equation: $\frac{dV}{dT} = \frac{4}{\cancel{3}} \cdot \pi \cdot \cancel{3} r^2 \frac{dr}{dT}$

$$\frac{9}{2}\,\frac{FT^3}{min} = 4\pi(2\,FT)^2 \frac{dr}{dT}$$

$$\frac{\frac{9\,FT^3}{2\,min}}{\frac{16\pi\,FT^2}{1}} = \frac{\cancel{16\pi\,FT^2}\,\frac{dr}{dT}}{\cancel{16\pi\,FT^2}}$$

$$\frac{9\,FT^{\cancel{3}}}{2\,min} \cdot \frac{1}{16\pi \cdot \cancel{FT^2}} = \frac{9}{32\pi}\,\frac{FT}{min}$$

Your Turn 1: Imagine a spherical balloon is being deflated at a constant rate of 10 cm^3 per second. How fast is the radius changing when the radius is 4 cm?

$$\frac{dV}{dT} = -10\ cm^3/sec$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dT} = \frac{4}{\cancel{3}} \cdot \pi \cdot \cancel{3} r^2 \frac{dr}{dT}$$

$$\frac{dV}{dT} = 4\pi r^2 \frac{dr}{dT}$$

$$-10\ cm^3/sec = 4 \cdot \pi \cdot (4\ cm)^2 \cdot \frac{dr}{dT}$$

$$\frac{-10\ \frac{cm^3}{sec}}{64\pi\ \cancel{cm^2}} = \frac{\cancel{64\pi \cdot cm^2} \cdot \frac{dr}{dT}}{\cancel{64\pi\ cm^2}}$$

$$\frac{-5}{32\pi}\ \frac{cm}{sec} = \frac{dr}{dT}$$

Example 7: Suppose you are drinking soda from a conical paper cup. The cup has diameter 8 cm and depth 10 cm. As you sip on the straw, soda leaves the cup at a rate of 7 cm^3/s. At what rate is the level of the liquid in the cup changing? When the liquid is 6 cm deep?

$h = 6$

$\frac{dh}{dt} = ?$

$\frac{dV}{dT} = -7 \frac{cm^3}{s}$

r = 4

h = 10 cm

Given:

Want:

Equation:

Since there are TWO unknowns, we need to replace 1 variable in terms of the other

$\frac{r}{h} = \cancel{8}\ \frac{4}{10}$

$4h = 10r$

$\frac{4}{10}h = \frac{2}{5}h = r$

Volume $\Rightarrow$ $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{2}{5}h\right)^2 \cdot h$

$V = \frac{1}{3}\pi \cdot \frac{4}{25} h^3$

$V = \frac{4\pi}{75} h^3$

$\frac{dV}{dT} = 3 \cdot \frac{4\pi}{75} \cdot h^2 \cdot \frac{dh}{dT}$

$-7 \frac{cm^3}{s} = \cancel{\frac{4\pi}{25}(6cm)^2} \cdot \frac{dh}{dT}$

$\frac{144\pi}{25} cm^2$ $\quad$ $\frac{\cancel{144\pi} cm^2}{25}$

$\frac{-175}{144\pi} \frac{cm}{sec} = \frac{dh}{dT}$

Example 8: Assume that the oil spilled from a ruptured tank spreads in a circular pattern whose radius increases at a constate of 2 ft/sec. How fast is the area of the spill increasing when the radius of the spill is 60 ft?

GIVEN	WANT
$\frac{dr}{dT} = 2\ \mathrm{FT}/\mathrm{sec}$	$\frac{dA}{dT} = ?$

$$\text{Area} = \pi r^2$$

$$\frac{dA}{dT} = 2\pi r \frac{dr}{dT}$$

$$2 \cdot \pi \cdot 60\ \mathrm{FT} \cdot 2\ \mathrm{FT}/\mathrm{sec}$$

$$\frac{dA}{dT} = 240\pi\ \mathrm{FT}^2/\mathrm{sec}$$

Example 9: The length of a rectangle if decreasing by 2 in/sec and the width is increasing by 3 in/sec . When the length is 10 inches and the width is 6 inches, how fast is the area changing?

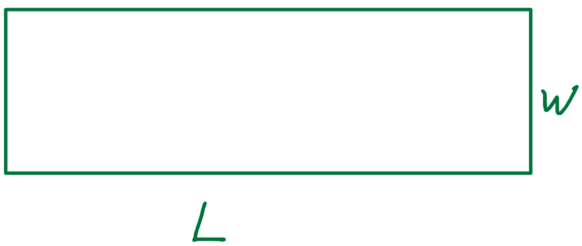
GIVEN

$\frac{dL}{dT} = -2\ in/sec$

$\frac{dW}{dT} = 3\ in/sec$

WANT

$\frac{dA}{dT}$



$$A = L \cdot W$$

$$\frac{dA}{dT} = \frac{dL}{dT} \cdot W + L \cdot \frac{dW}{dT}$$

$$\frac{dA}{dT} = -2\ in/sec \cdot 6\ in + 10\ in \cdot 3\ in/sec$$

$$\frac{dA}{dT} = -12\ \frac{in^2}{sec} + 30\ in^2/sec$$

$$\frac{dA}{dT} = 18\ in^2/sec$$

RELATED RATE DEMO

The situation.

A ladder 10 ft in length is leaning against a brick wall.
The top of the ladder is originally 8.5 ft high. The top of the ladder falls at a fixed rate of speed dy/dt.
As time goes by the distance x(t) from the base of the wall to the bottom of the ladder changes.
WHAT is the RATE of CHANGE of the distance x(t)?

Find dx/dt.

x(t) y(t)

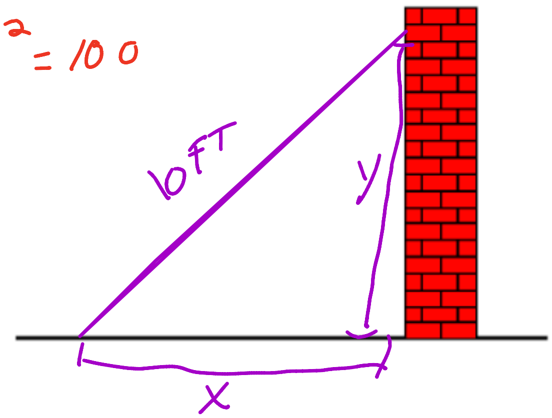
$y = 8.5$

$\frac{dx}{dt}$

Units are in feet.

$(8.5)^2 + x^2 = 100$

$x \approx 5.3$



$x^2 + y^2 = 10^2$

$2x\frac{dx}{dT} + 2y\frac{dy}{dT} = 0$

$2(5.3)\frac{dx}{dT} + 2(8.5)\frac{dy}{dT} = 0$

$\frac{\cancel{10.6}\frac{dx}{dT}}{\cancel{10.6}} = \frac{-17\frac{dy}{dT}}{10.6}$

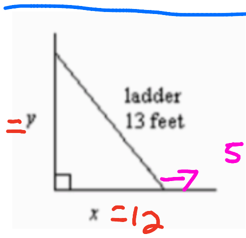
$\frac{dx}{dT} = \frac{-17}{10.6} \cdot \frac{dy}{dT}$

$x^2+y^2=13^2$

Example 9.5: A 13-ft ladder is leaning against a wall when its base starts to slide away from the wall. By the time its base is 12 ft from the wall, the base is moving at the rate of 5 ft/sec. How fast is the tip of the ladder sliding down the wall then?

$\frac{dx}{dT}$

$5 =$



$=12$

5 FT/sec

$\frac{dy}{dT}$

$$x^2+y^2=169$$

$$2x\frac{dx}{dT}+2y\frac{dy}{dT}=0$$

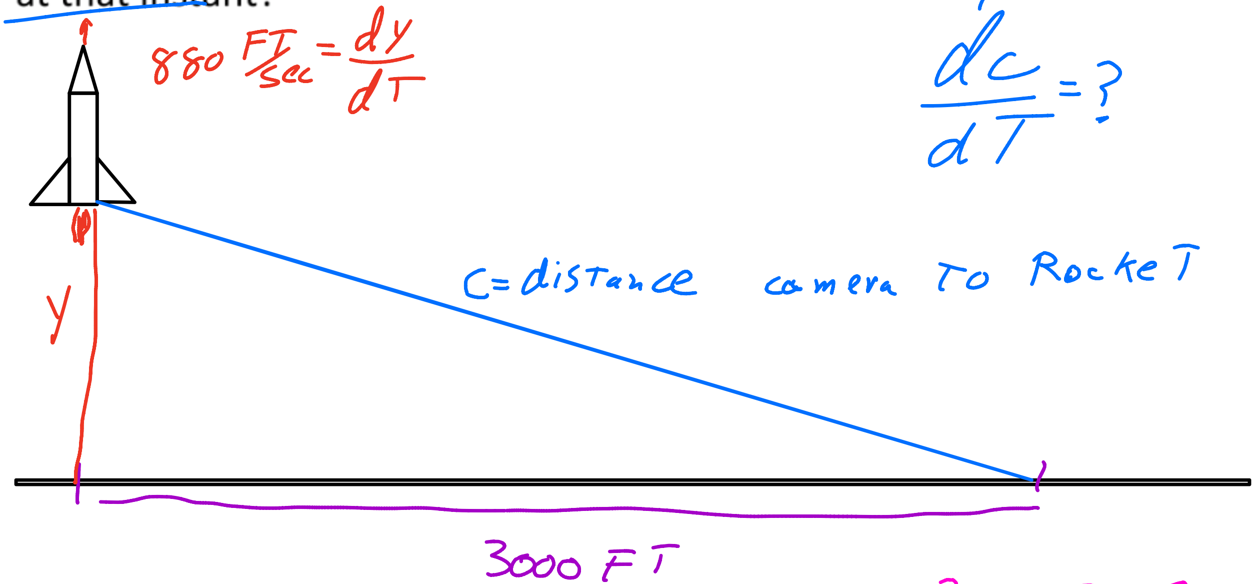
$$2(12\,FT)\left(5\,\frac{FT}{S}\right)+2(5\,FT)\frac{dy}{dT}=0$$

$$120\,FT^2/S+10\,FT\cdot\frac{dy}{dT}=0$$

$$\frac{\cancel{10\,FT}\cdot\frac{dy}{dT}}{\cancel{10\,FT}}=\frac{-120\,FT^2/S}{10\,FT}$$

$$\frac{dy}{dT}=-12\;FT/S$$

Example 10: A camera is placed 3000 feet from a rocket launching pad . If a rocket is rising vertically at 880 feet per second when it is 4000 feet in the air, how fast is a camera-to-rocket distance changing at that instant?



$$4000^2 + 3000^2 = c^2$$
$$5000 = C$$

$$y^2 + 3000^2 = c^2$$

$$2y\frac{dy}{dT} + 0 = 2c\frac{dc}{dT}$$

$$2(4000\,FT)(880\,FT/sec) = 2 \cdot 5000\,FT \cdot \frac{dc}{dT}$$

$$\frac{7040000\ FT^2/sec}{10{,}000\ \cancel{FT}} = \frac{\cancel{10{,}000\,FT}\ \frac{dc}{dT}}{\cancel{10{,}000\,FT}}$$

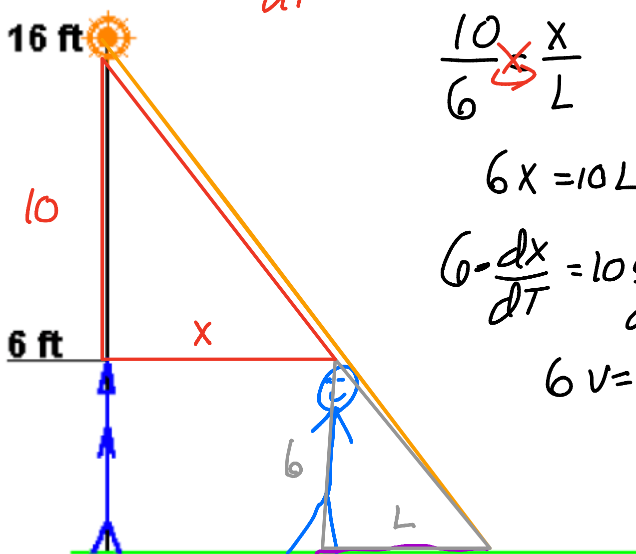
$$704\ FT/sec = \frac{dc}{dT}$$

RELATED RATE DEMO

The situation.

A man 6 ft tall is directly under a light which is 16 feet high.
He walks to the right at a steady speed of v ft/sec. $V = \frac{dx}{dT}$
As time goes by the length of his shadow increases.

$\frac{dL}{dT}$



Shadow = L

$$\frac{10}{6} = \frac{x}{L}$$

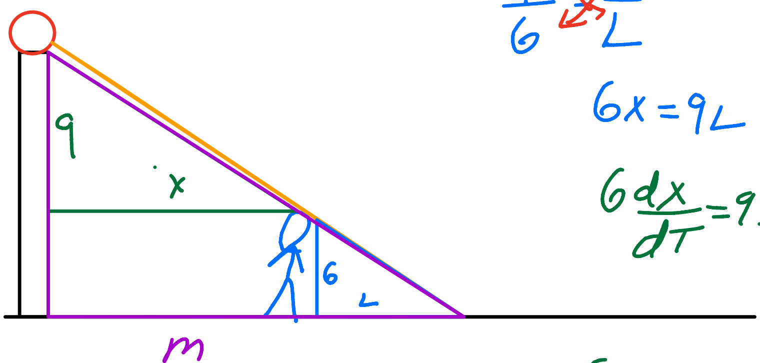
$$6x = 10L$$

$$6 \cdot \frac{dx}{dT} = 10\frac{dL}{dT}$$

$$6v = 10 \text{ ft} \frac{dL}{dT}$$

$$\frac{6}{10}v = \frac{dL}{dT}$$

Example 12: A street light is mounted at the top of a 15 ft. pole. A 6 ft. tall man walks away from the pole at a rate of 5 feet per second. How fast is the length of his shadow moving when he is 40 feet from the pole? How fast is the tip of his shadow moving?



$\frac{dx}{dT} = 5\,FT$

$\frac{9}{6} = \frac{x}{L}$

$6x = 9L$

$6\frac{dx}{dT} = 9\frac{dL}{dT}$

$6 \cdot 5 = 9\frac{dL}{dT}$

$3\frac{1}{3}\,FT/sec = \frac{10}{3}\,FT/sec = \frac{dL}{dT}$

$\frac{15}{m} = \frac{9}{x}$

$9m = 15x$

$9\frac{dm}{dT} = 15\frac{dx}{dT}$

$9\frac{dm}{dT} = 75\,FT/S$

$\frac{dm}{dT} = \frac{75}{9}\,FT/S = 8\frac{3}{9}\,FT/sec = 8\frac{1}{3}\,FT/sec$